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A NEW METHOD FOR SOLVING DODECAGONAL FUZZY ASSIGNMENT PROBLEM

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Abstract: Assignment problem is a special case of linear programming problem. The objective of the optimal assignment is to minimize the total cost or maximize the profit. Fuzzy set theory has been applied in many fields of science, Engineering and Management. In this paper a new ranking method is proposed for solving the dodecagonal fuzzy assignment problem. Fuzzy assignment problem transformed into crisp assignment problem and solved by Hungarian method. A numerical example is presented and the optimal solution is derived by using proposed method.

Keywords and Phrases: Dodecagonal Fuzzy number, Ranking Method, Fuzzy Assignment Problem.

2010 Mathematics Subject Classification: 03E72, 90B06, 90C70.

1. Introduction

Assignment problem is the special case of linear programming problem. Assignment problem can be applied in all fields like Science, Engineering, and Management etc. Assignment problem plays an important role in industry and other applications. In assignment problem " n "jobs are assigned " n "persons depending their on their efficiency to do the job. The objective of the optimal assignment is to minimize the total cost or maximize the profit. Fuzzy assignment problems have received great attention in recent years. Here we investigate a more realistic problem, namely the assignment problem with fuzzy costs or times. The objective is to minimize the cost or to maximize the total profit, subject to some crisp constraints, the objective function is considered also as a fuzzy number.

L. A. Zadeh [20] introduced the concept of fuzzy sets to deal with imprecision, vagueness in real life situations. R. E. Bellmann and L. A. Zadeh [2] proposed the concept of decision making in fuzzy environment. C. J. Lin and U. P. Wen [8] was finding the solution of fuzzy assignment problem using labeling algorithm. H. Basirzadeh [1] approached a new technique for solving fuzzy transportation problem. M. S. Chen [3] proved some theorems and proved a fuzzy assignment model that considers all individuals to have same skills.

Amit Kumar and Anila Gupta [7] were first solved fuzzy assignment problem and travelling salesman problem with different membership function. R. Jahirhussain and P. Jayaraman [5] solved fuzzy assignment problem using robust ranking method. T. S. Pavithra and C. Jenita [14] proposed a new a method for solving a dodecagonal fuzzy assignment problem. S. Manimaran and M. Ananthanarayanan [9] were discussed a comparative study of two fuzzy numbers using average method. Y. L. P. Thorani and N. Ravi Shankar [19] were studied the applications of fuzzy assignment problems. D. Stephen Dinagar and S. Kamalanathan [16] solved a fuzzy assignment problem with two different ranking methods. L. Sujatha and S. Elizabeth [17] solved the fuzzy transportation problem and fuzzy unbalanced assignment problem by using one point method. Anchal Choudhary, R. N. Jat, C. Sharma and Sanjay Jain [4] were applied a branch and bound technique for solving fuzzy assignment problem.

P. Pandian and K. Kavitha [13] solved the assignment problems using parallel moving method. A. Nagoor Gani and V. N. Mohamed [12] proposed a new method for solving assignment problem for trapezoidal fuzzy numbers. Sunil Kumar Mehta, Neha Ishesh Thakur, Parmpreet Kaur [10] had approached numerical methods to find the solution of fuzzy assignment problem. Y. L. P. Thorani and N. Ravi Shankar [18] solved fuzzy assignment problem with generalized fuzzy number. Supriya Kar, KajlaBasu, Sathi Mukherjee [6] finding solution of generalized fuzzy assignment problem with restriction on the cost of both job and person under fuzzy environment. S. Muruganandhan and D. Hema [11] solved fuzzy assignment method using one suffix method. Jatinder Pal Singh and Neha Ishesh Thakur [15] were solved fuzzy transportation problem using dodecagonal fuzzy number.

In this paper a new ranking method is proposed in solving a dodecagonal fuzzy assignment problem. Fuzzy assignment problem can be converted into crisp assignment problem using ranking method and an optimal solution is obtained by using Hungarian method.

2. Preliminaries

Definition 2.1. *A fuzzy set is characterized by a membership function mapping element of a domain space or the universe of discourse X to the unit interval $\{0, 1\}$.*

$$(i.e)A = \{x, \mu_A(x); x \in X\}, \quad here \ \ \mu_A(x) = 1$$

Definition 2.2. *A fuzzy set A of the universe of discourse X is called normal fuzzy set implying that there exist atleast one such that $\mu_A(x) = 1$.*

Definition 2.3. *support of a fuzzy set in the universal set X is the set contains all the elements of X that have a non-zero membership grade in $\widetilde{A}$.*

$$Supp(\widetilde{A}) = \{x \in X \mid \mu_{\widetilde{A}}(x) > 1\}$$

Definition 2.4. *Given a fuzzy set A defined on X and any number $\alpha \in [0, 1]$ the $\alpha-$cut, α_A is the crisp set,*

$$\alpha_A = \{x \in X \mid A_x \geq \alpha, \alpha \in [0, 1]\}$$

Definition 2.5. *A fuzzy set $\widetilde{A}$ defined on the set of real numbers R is said to be fuzzy number if its membership function $\mu_A : R \to [0, 1]$ has the following properties,*
1. A must be a normal and convex fuzzy set.
2. α_A must be a closed interval for every $\alpha \in [0, 1]$.
3. The support of $\widetilde{A}$ must be bounded.

Definition 2.6. *A fuzzy number $\widetilde{A}$ is called triangular function is denoted by $\widetilde{A} = (a_1, a_2, a_3)$ whose membership function is defined as follows,*

$$\mu_{\widetilde{A}}(x) = \begin{cases} 0 & x < a_1 \\ \frac{x-a_1}{a_1-a_2} & a_1 \leq x \leq a_2 \\ \frac{a_3-x}{a_3-a_2} & a_2 \leq x \leq a_3 \\ 0 & x > a_3 \end{cases}$$

Definition 2.7. *A fuzzy number $\widetilde{A}$ is called trapezoidal function is denoted by*

$\widetilde{A} = (a_1, a_2, a_3, a_4)$ *whose membership function is defined as follows,*

$$\mu_{\widetilde{A}}(x) = \begin{cases} 0 & x{<}a_1 \\ \frac{x-a_1}{a_1-a_2} & a_1 \leq x \leq a_2 \\ 1 & a_2 \leq x \leq a_3 \\ \frac{a_4-x}{a_4-a_3} & a_3 \leq x \leq a_4 \\ 0 & x > a_3 \end{cases}$$

3. Dodecagonal Fuzzy Number

Definition 3.1. *A fuzzy number* $\widetilde{A}$ *is a Dodecagonal fuzzy number defined by* $\widetilde{A}$=(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12})*, where* a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12} *are real numbers and its membership function is given by,*

$$\mu_{\widetilde{A}}(x) = \begin{cases} 0 & x \leq a_1 \\ k_1\left(\frac{x-a_1}{a_2-a_1}\right) & a_1 \leq x \leq a_2 \\ k_1 & a_2 \leq x \leq a_3 \\ k_1 + (k_2-k_1)\left(\frac{x-a_3}{a_4-a_3}\right) & a_3 \leq x \leq a_4 \\ k_2 & a_4 \leq x \leq a_5 \\ k_2 + (1-k_2)\left(\frac{x-a_5}{a_6-a_5}\right) & a_5 \leq x \leq a_6 \\ 1 & a_6 \leq x \leq a_7 \\ k_2 + (1-k_2)\left(\frac{a_8-x}{a_8-a_7}\right) & a_7 \leq x \leq a_8 \\ k_2 & a_8 \leq x \leq a_9 \\ k_1 + (k_2-k_1)\left(\frac{a_{10}-x}{a_{10}-a_9}\right) & a_9 \leq x \leq a_{10} \\ k_1 & a_{10} \leq x \leq a_{11} \\ k_1\left(\frac{a_{12}-x}{a_{12}-a_{11}}\right) & a_{11} \leq x \leq a_{12} \\ 0 & a_{12} \leq x \end{cases}$$

for $0 < k_1 < k_2 < 1$.

3.1. Arithmetic Operations on Dodecagonal Fuzzy Number

Let $\widetilde{A}_{DDFN}$=(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}) & $\widetilde{B}_{DDFN}$= (b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8, b_9, b_{10}, b_{11}, b_{12}), be two dodecagonal fuzzy numbers then the addition, subtraction and scalar multiplications can be performed as follows,
$\widetilde{A}_{DDFN}$+ $\widetilde{B}_{DDFN}$=[$a_1 + b_1$, $a_2 + b_2$, $a_3 + b_3$, $a_4 + b_4$, $a_5 + b_5$, $a_6 + b_6$, $a_7 + b_7$, $a_8 + b_8$, $a_9 + b_9$, $a_{10} + b_{10}$, $a_{11} + b_{11}$, $a_{12} + b_{12}$]

$\widetilde{A}_{DDFN}$ - $\widetilde{B}_{DDFN}$=$[a_1 - b_{12},\ a_2 - b_{11},\ a_3 - b_{10},\ a_4 - b_9,\ a_5 - b_8,\ a_6 - b_7,\ a_7 - b_6,\ a_8 - b_5,\ a_9 - b_4,\ a_{10} - b_3,\ a_{11} - b_2,\ a_{12} - b_1]$

$\lambda\widetilde{A}_{DDFN}$=$[\lambda a_1,\ \lambda a_2,\ \lambda a_3,\ \lambda a_4,\ \lambda a_5,\ \lambda a_6,\ \lambda a_7,\ \lambda a_8,\ \lambda a_9,\ \lambda a_{10},\ \lambda a_{11},\ \lambda a_{12}]$

$\lambda\widetilde{B}_{DDFN}$= $[\ \lambda b_1,\ \lambda b_2,\ \lambda b_3,\ \lambda b_4,\ \lambda b_5,\ \lambda b_6,\ \lambda b_7,\ \lambda b_8,\ \lambda b_9,\ \lambda b_{10},\ \lambda b_{11},\ \lambda b_{12}]$

3.2. Measure of Fuzzy Number

The measure of $\widetilde{A}_\omega$ is a measure is a function $M_o : R_\omega(I) \to R^+$ which assign a non-negative real numbers $M_0^{DDFN}(\widetilde{A}_\omega)$ that expresses the measure of

$$M_0^{DDFN}(\widetilde{A}_\omega) = \frac{1}{2}\int_{\alpha}^{k_1}\left(f_1(r) + \bar{f}_1(r)\right)dr + \frac{1}{2}\int_{k_1}^{k_2}\left(g_1(s) + \bar{g}_1(s)\right)ds + \frac{1}{2}\int_{k_2}^{\omega}\left(h_1(t) + \bar{h}_1(t)\right)dt \tag{3.1}$$

where $0 \le \alpha < 1$.

4. Proposed Ranking Method

Let $\widetilde{A}$ be a normal dodecagonal fuzzy number. The measure of $\widetilde{A}$ is defined by,

$$M_0^{DDFN}(\widetilde{A}_\omega) = \frac{1}{2}\int_{\alpha}^{k_1}\left(f_1(r) + \bar{f}_1(r)\right)dr + \frac{1}{2}\int_{k_1}^{k_2}\left(g_1(s) + \bar{g}_1(s)\right)ds + \frac{1}{2}\int_{k_2}^{\omega}\left(h_1(t) + \bar{h}_1(t)\right)dt$$

$$M_0^{DDFN}(\widetilde{A}) = \frac{1}{4}\{(a_1 + a_2 + a_{11} + a_{12})k_1 + (a_3 + a_4 + a_9 + a_{10})(k_2 - k_1) + (a_5 + a_6 + a_7 + a_8)(1 - k_1)\} \tag{4.1}$$

where $0 < k_1 < k_2 < 1$

5. Mathematical Formulation of Fuzzy Assignment Problem

The assignment problem can be stated in the form of cost matrix of fuzzy numbers as follows:

Persons	**Jobs**			
	1	**2**	...	**n**
1	$\widetilde{c}_{11}$	$\widetilde{c}_{12}$		$\widetilde{c}_{1n}$
2	$\widetilde{c}_{21}$	$\widetilde{c}_{22}$		$\widetilde{c}_{2n}$
...	...	...	...	...
n	$\widetilde{c}_{11}$	$\widetilde{c}_{12}$		$\widetilde{c}_{nn}$

The mathematical formulation of the fuzzy assignment problem is given by

$$minimize\ \widetilde{Z}^* = \sum_{i=1}^{n}\sum_{j=1}^{n}\widetilde{c}_{ij}x_{ij}^*$$

Subject to the constraints $\sum_{i=1}^{n} x_{ij} = 1, i = 1, 2, ..., n$ $\sum_{j=1}^{n} x_{ij} = 1, j = 1, 2, ..., n$ where x_{ij} is the decision variable
and

$$x_{ij} = \begin{cases} 1 & \text{if the } i^{th} \text{ person assigned to } j^{th} \text{ job} \\ 0 & \text{otherwise} \end{cases} \tag{5.1}$$

$\widetilde{c}_{ij}$ is the fuzzy assignment cost of i^{th} job to j^{th} person. Hence it cannot be solved directly. For solving the problem, we first defuzzify the fuzzy cost coefficients into crisp ones by the above fuzzy number ranking method (4.1).

5.1. Numerical Example

Consider the following dodecagonal fuzzy assignment problem which consists of four jobs and four machines. The cost matrix $\widetilde{c}_{ij}$ whose costs are dodecagonal fuzzy numbers. Here our objective is to find the optimum assignment so as to minimize the cost (or time).

Table 1: Dodecagonal Fuzzy Assignment Problem

	W$_1$	**W**$_2$	**W**$_3$	**W**$_4$
J$_1$	(1,3,5,7,9,11,13, 15,17,19,21,23)	(2,4,6,8,10,12,14, 16,18,20,22,24)	(1,2,3,4,5,6,7, 8,9,10,11,12)	(1,2,3,4,5,7,9, 11,13,17,21,25)
J$_2$	(3,7,11,13,17,21, 22,25,29,32,40,43)	(2,4,6,8,9,13,15, 16,18,20,21,25)	(2,3,7,8,9,11,13, 15,16,21,33,37)	(1,3,5,7,9,12,15, 18,21,25,29,33)
J$_3$	(1,2,3,4,7,10,13, 15,16,17,22,26)	(5,8,10,13,16,21, 23,28,31,32,37,39)	(4,6,7,9,10,11,18, 23, 24,26,27,30)	(2,3,5,7,10,13,17, 21,25,29,34,39)
J$_4$	(2,3,5,7,11,13,17, 19,23,29,31,35)	(1,2,3,4,7,10,13, 16,20,24,28,32)	(1,4,7,10,13,16,19, 22,25,28,31,34)	(3,6,9,12,16,20,24, 29,33,38,40,42)

5.2. Ranking of Dodecagonal Fuzzy Number

In order to find the optimum value of the given dodecagonal fuzzy cost given in table 1, first we convert the fuzzy cost into the crisp cost using the proposed ranking method (4.1). Take the values of $k_1 = 0.3, k_2 = 0.7$. The ranking of fuzzy numbers is done by using proposed ranking method (4.1). The crisp assignment problem of the corresponding dodecagonal fuzzy assignment problem is given in the following table. The given fuzzy assignment problem is a balanced assignment problem. By applying the Hungarian method, we find the optimal assignment schedule and the optimum assignment cost. The optimal assignment schedule is given by $J_1 \to W_3,\ J_2 \to W_4,\ J_3 \to W_1,\ J_4 \to W_2$. The optimal assignment cost =

Table 2: Crisp assignment problem of the corresponding Dodecagonal fuzzy assignment problem

	$\mathbf{W}_1$	$\mathbf{W}_2$	$\mathbf{W}_3$	$\mathbf{W}_4$
$\mathbf{J}_1$	12	13	6.5	9.8
$\mathbf{J}_2$	21.9	13.1	14.4	14.9
$\mathbf{J}_3$	11.2	21.8	61.3	17
$\mathbf{J}_4$	16.2	13.3	17.5	22.7

$6.5+14.9+11.2+13.3 = 45.9$ units

6. Conclusion

Ranking of fuzzy number plays an important role in decision making problems and some other fuzzy application system. Fuzzy numbers must be ranked before an action is taken by a decision maker. Ranking methods which convert a fuzzy number to a crisp number by applying a mapping function. In this paper, a new method is proposed for solving dodecagonal fuzzy assignment problem. The proposed ranking method is simple and easy to calculate rank of fuzzy numbers which also gives perfect solution to the given problem. This ranking method is used to rank the all the Dodecagonal fuzzy numbers. The advantage of the proposed model is illustrated by examples. In future, this method is applied to assigning jobs to suitable persons in a real life problem.

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